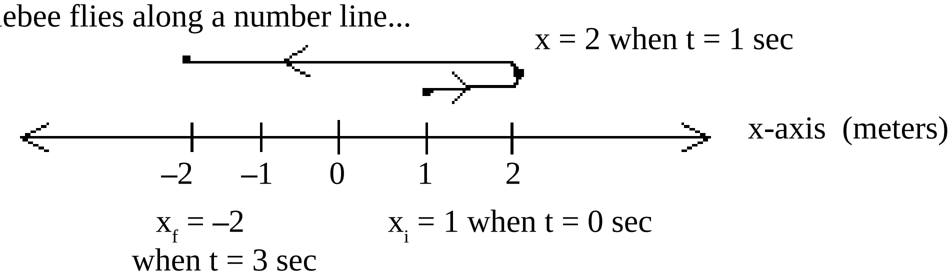
KINEMATICS IN ONE DIMENSION

Motion involves a change in position. <u>Position</u> can be indicated by an <u>x-coordinate</u> on a number line.

ex/ A bumblebee flies along a number line...



<u>DISPLACEMENT</u> or <u>change in position</u> is defined as: $\Delta x \equiv x_f - x_i$

ex/ Here displacement or $\Delta x = x_f - x_i = {}^-2 - 1 = {}^-3(meters\ or\ m)$

<u>DISTANCE TRAVELED</u> is always positive and all the zig-zags must be added up.

ex/ Here distance traveled equals 1+4 or 5m

<u>Distance from the starting point</u> or from the initial position is also positive but a different concept still.

ex/ Here distance from the initial position is $|x_f - x_i|\ or\ |x_i - x_f|\ which\ is\ |{}^-2 - 1| = 3m$

<u>AVERAGE SPEED</u> (recall d = rt from first year algebra) is defined as: $\dfrac{\text{distance traveled}}{\text{elapsed time}}$

Let's define elapsed time as $\Delta t = t_f - t_i$ (always positive).

ex/ Here average speed (it doesn't have a convenient letter, although 'r' was used in algebra)

equals: $\dfrac{1+4}{\Delta t} = \dfrac{5m}{3-0\text{ sec}} = 1.67\dfrac{m}{s}(correct\ to\ three\ significant\ \ figures)$

Now if the bumblebee was really zig-zagging back and forth like an electron in a live wire, we might be more interested in the displacement made by the bee in the 3 seconds it was moving...

<u>AVERAGE VELOCITY</u> is defined as: $\bar{v} = v_{AVG} \equiv \dfrac{\Delta x}{\Delta t}$

ex/ Here average velocity equals:

$\bar{v} = \dfrac{{}^-2-1}{3-0} = {}^-1\dfrac{m}{s}$ (On the average, the bee moved in the negative direction 1m every second.)

Notice that <u>average velocity</u>, unlike average speed, can be <u>negative</u>. Average velocity is a <u>vector</u> concept in that it has <u>direction</u> (in this case, negative) as well as <u>magnitude</u> (1 m/s).

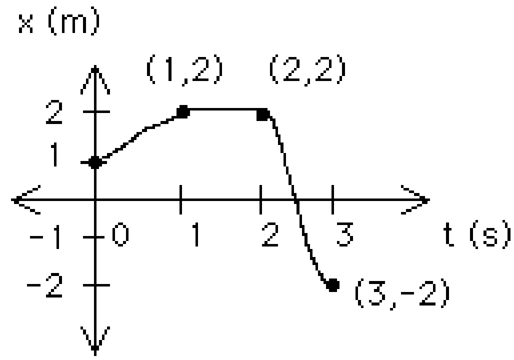
<u>Displacement</u>, unlike distance traveled, can be <u>negative</u>. Displacement, like average velocity, is a <u>vector</u> concept with a <u>direction</u> (in this case, negative) and a <u>magnitude</u> (3 m).

Notice also that average speed is <u>not the absolute value</u> of average velocity, just as distance traveled is <u>not the absolute value</u> of displacement. Average speed (1.67 m/s) is not the magnitude (1 m/s) of average velocity (-1 m/s).

POSITION-TIME GRAPHS

Let's plot some points on a graph to show where our bumblebee was from t = 0s to t = 3s.

t (s)	x (m)
0	1
1	2
2	2
3	-2



ex / Find $\bar{v}$ for the first second of travel. $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{2 - 1\text{m}}{1 - 0\text{s}} = 1\frac{m}{s}$

Locate the two relevant black dots on the graph. If we drew a secant line through these two points and calculated the slope, we would find that the 'rise over run' would be $1\frac{\text{m}}{\text{s}}$.

(In algebra, slope or m = $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$.)

ex / Find $\bar{v}$ over the time interval from t = 1sec to t = 2sec. $\bar{v} = m_{\text{secant line}} = 0\frac{\text{m}}{\text{s}}$.

Connect the two points. What is the slope of a horizontal line? (answer: zero)

ex / Find $\bar{v}$ for the time interval, $0 \le t \le 3\text{s}$. $\bar{v} = \frac{\text{rise}}{\text{run}} = \frac{-3m}{3 - 0s} = -1\frac{m}{s}$. Draw the secant line through the correct two points. Is the slope of this line positive or negative? (answer: negative)

ex / What is the distance traveled from t = 1 to t = 2s ?

ex / What is the average speed from t = 1 to t = 2s ? Which direction is the bumblebee moving during this time interval?

When the graph goes up or has a positive slope the bumblebee is moving to the right on the the x-axis.
When the graph goes down or has a negative slope the bumblebee is moving to the left on the x-axis.
What is the position or location of the bee as indicated by the ordered pair, (1,2)? (answer: x=2m)
Where is the bumblebee when t=3s ? (answer: x=-2m and no credit for writing '(3,-2)')
The point to make here is that the graph is not a path of the bumblebee journey in 2-dimensions. The position-time graph is a visual representation of some data. The slope is not always the steepness of some hill. What, then, is the meaning of slope?

The slope of a graph in 2-dimensions is the rate of change of one variable (here, x or position) with respect to another variable (here, t or time). This average rate of change is called average velocity here. It need not have a special name. A slope of a '$ vs time' graph might represent how much the value of your comic book collection changes from year to year. The slope of a 'meter vs kilometer' graph could be constant, 1000m for every 1km or a slope of 1000 m/km. The units of our slope become crucial. Without them, what meaning would you give to a slope of 3?

THE POSITION FUNCTION

Suppose we have a mathematical function which can tell us the position of a particle at any instant in time. We call such a function, a position function, and if the position is given by an x-coordinate, we write:

$x = f(t)$ (Careful! This is like writing $y=f(x)$.)

ex/ Let's look at a specific function without the 'f',

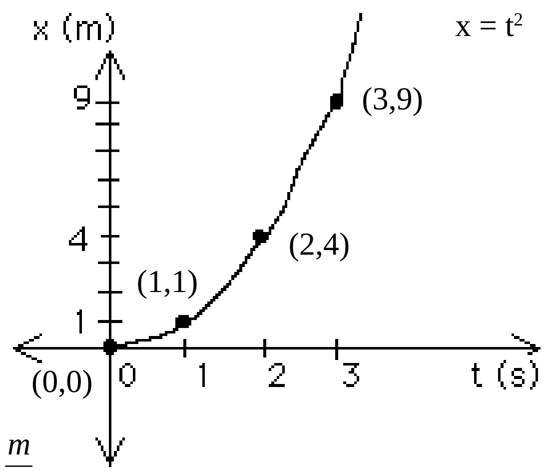
$x = t^2$

or

$x(t) = t^2$

(new notation, read "x of t equals t-squared")

position-time graph



ex / What is $\bar{v}$ for the first second of travel? $\bar{v} = \dfrac{1 - 0\text{m}}{1 - 0\text{s}} = 1\dfrac{m}{s}$.

Draw the secant line thru (0,0) and (1,1). What is the slope of this line?

ex / What is $\bar{v}$ for the time interval, $1 \le t \le 3\text{s}$? $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{9-1}{3-1} = 4\dfrac{m}{s}$

Draw the secant line thru (1,1) and (3,9). What does the slope of this line mean to you?

Instantaneous Velocity (derivative of the position function, slope of the tangent line on a position-time graph, instantaneous rate of change of position w.r.t. time)

Consider the following question. How fast is the bumblebee going at the instant t=1sec ?
If only we could have seen what its speedometer read at that instant...
Which is a better guess, 1 m/s or 4 m/s ? Why?

ex / Why should we compute the average velocity from $t = \dfrac{1}{2}s$ to $t = 1\text{s}$?

What is the slope of the secant line thru $(\dfrac{1}{2}, \dfrac{1}{4})$ and (1,1)? $\bar{v} = \dfrac{1 - \frac{1}{4}}{1 - \frac{1}{2}} = \dfrac{3/4}{1/2} = 1.5\dfrac{m}{s}$.

ex / What is the slope of the secant line thru (.9,.81) and (1,1)? $\bar{v} = \dfrac{1\text{-}.81\text{m}}{1\text{-}.9\text{s}} = \dfrac{.19}{.1} = 1.9\dfrac{m}{s}$.

Draw the secant line thru these two points. What is your guess about the slope of the tangent line thru the one point (1,1) ? (answer: $2\dfrac{\text{m}}{\text{s}}$)

The derivative of the graph of y=f(x) gives the slope of that tangent line. In calculus, we discover the 'power rule'. The derivative of power functions such as $y = x^n$ is given by: $y' = f'(x) = \dfrac{dy}{dx} = nx^{n-1}$

$$y = x^7 \Rightarrow y' = 7x^6,\ y = 3x^4 - x^3 + 4x^2 \Rightarrow f'(x) = 12x^3 - 3x^2 + 8x,\ y = 3x + 2 \Rightarrow \frac{dy}{dx} = 3,\ y = 3.5 \Rightarrow m_t = 0$$

INSTANTANEOUS VELOCITY is the derivative of the position function with respect to time: $v \equiv \dfrac{dx}{dt}$.

ex / If $x=t^2$ then $v(t)=2t$. Hence, $v(1)=2$ m/s. What does 'v(3)' mean? $v(t)=2t$, therefore $v(3)=6$ m/s.

INSTANTANEOUS SPEED is the magnitude or absolute value of instantaneous velocity: $|v(t)| = \left|\frac{dx}{dt}\right|$

ex / Consider the following position function of a helicopter moving up and down only:

$y(t) = 4t^2 - 16t + 100$ (assume SI or MKS units if not indicated)

(a) What is the instantaneous velocity when t=0sec ?
(b) What is the instantaneous speed when t=0sec ?
(c) Write the velocity function for this helicopter.
(d) Where is the helicopter when t=3sec ?
(e) In what direction is the helicopter headed when t=3sec ?
(f) When is the velocity (instantaneous) zero? What is happening at this instant?
(g) What is the speed (instantaneous) when t=3sec ?
(h) What is the average speed from t=0 to t=3sec ?

Answers:

(a) v(0) = -16 m/s
(b) $|v(0)| = 16\ m/s$
(c) v(t) = 8t – 16
(d) y(3) = 88 m
(e) upward or positive-y
(f) v = 8t – 16 = 0 at t = 2s
A turning point? At rest?
(g) $|v(3)| = |24 - 16| = 8\ m/s$
(h) This one is messy!

$speed_{AVG} = \frac{|y(2) - y(0)| + |y(3) - y(2)|}{3\text{sec}}$

$= \frac{16 + 4}{3} = 6.67\ m/s$

AVERAGE ACCELERATION is defined as: $\bar{a} = a_{AVG} \equiv \frac{\Delta v}{\Delta t}$

where change in velocity or $\Delta v = v_f - v_i$

a_{AVG} will be the slope of a secant line on a velocity-time graph.

ex / If v_i = -16 m/s and v_f = 8 m/s from t = 0s to t = 3s, then
a_{AVG} = 8 – ⁻16 / 3s = 8 m/s^2

Acceleration units need to be studied closely. What does '8 m/s^2' mean?

Think "velocity changes $^{+}8\frac{m}{s}$ per sec" or "$8\frac{\text{m/s}}{s}$".

We write $8\frac{\text{m}}{\text{s}^2}$.

INSTANTANEOUS ACCELERATION is defined as: $a = a(t) \equiv \frac{dv}{dt}$

Instantaneous acceleration will be the slope of a tangent line on a velocity-time graph

Consider a butterfly moving back and forth along a number line according to:

$\text{x(t)} = 2\text{t}^3 - 5\text{t}^2 + 7\text{t} + 100,\ \text{m}$ (position function)

$\text{v(t)} = 6\text{t}^2 - 10\text{t} + 7,\ m/s$ (velocity function)

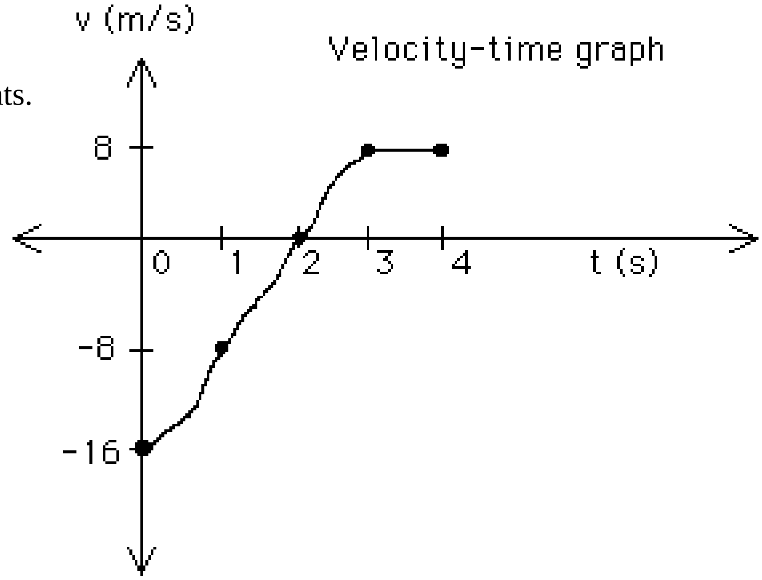
$\text{a(t)} = 12\text{t} - 10,\ \text{m/s}^2$ (acceleration function)

ex / (a) Where is the butterfly at t=1s ? Is it moving left or right?
(b) What is v(1) ? What is the butterfly's acceleration at t=1s ?
(c) What is v(0) ? What is a_{AVG} over the time interval from t=0 to t=1s ?

VELOCITY-TIME GRAPHS

Let's graph some velocity-time data points.

t (s)	v (m/s)
0	-16
1	-8
2	0
3	8
4	8



ex / Assume the data and graph refer to the motion of a butterfly moving left and right on the x-axis.

(a) Which way is the butterfly moving at the instant t = 1s ?
(b) What is the speed of the butterfly when t = 0s ?
(c) Over what interval of time is the butterfly moving to the right?
(d) When is the butterfly at the origin (where x = 0m)?
(e) When is the butterfly at rest?How long is the butterfly resting?
(f) What is the average acceleration of the butterfly from t = 0 to t = 2s ?
(g) Where is the butterfly when t = 3s ?
(h) Over what time interval or at what instant(s) in time is acceleration zero?
(i) When is the butterfly moving to the left? When is it moving upward?
(j) Find a_{AVG} for the time interval from t = 1 to t = 4s. Draw the secant line on the graph.
(k) Using tangent lines, approximately when was acceleration the greatest?

Answers:

(a) Since v(1) = -8 m/s, the butterfly is moving left.
(b) Since v(0) = -16 m/s, instantaneous speed is +16 m/s.
(c) Moving to the right means a positive velocity, from t=2 to t=4s.
(d) We don't know since we don't have a position graph or function.
(e) 'At rest' (in physics) means instantaneou velocity is zero, at t = 2s.
(f) Did you draw the secant line thru (0,-16) and (2,0)? slope = 8 m/s^2.
(g) We don't know!
(h) Slope is zero from t = 3 to t = 4s.
(i) Moving left means a negative velocity, from t =0 to t = 2s.
The butterfly is not moving up or down in this problem.
(j) What is the slope of the secant line thru (1,-8) and (4,8)? 5.33 m/s^2.
(k) It looks like the steepest tangent line occurs when t = .8s ?

WARNING: Many words like acceleration, speed , and velocity have different English vs physics meanings.

In English, velocity and speed are synomyms. In physics, velocity and speed are different concepts.
In English, acceleration means to "speed up". In English, deceleration means to "slow down".
In physics, consider...

ex / What does a constant acceleration of -5 m/s^2 mean? Speeding up or slowing down?
Neither! (i)If at this instant your car has a velocity of 22 m/s, then 1sec later v = 17 m/s.
(ii)If at this instant your car has a velocity of -13 m/s, then 1sec later v = -18 m/s.
In case (i) your **speed** decreased. In case (ii) your **speed** increased from 13 to 18 m/s.
In both cases your **velocity** changed in the negative direction or decreased (algebraically).